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The Effect of Kronecker Tensor Product Values on ECG Rates: A Study on Savitzky-Golay Filtering Techniques for Denoising ECG Signals

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ABSTRACT

This article presents a study on ECG signal filtering algorithms to denoise signals corrupted by various types of noise sources. The study also examines the effect of Kronecker tensor product values on ECG rates. The study is conducted in a Matlab environment, and the results demonstrate that a constant number for the respective codes can effectively denoise ECG signals without any trouble. These findings have significant implications for diagnosing abnormal heart rhythms and investigating chest pains. The present study is novel in that it explores the relationship between ECG rate and Kronecker delta values across different age groups, which has not been extensively studied in previous literature. The study's unique contribution is the determination of age-specific values of the constant K required to represent this relationship accurately in different populations, which could inform the development of more effective algorithms for denoising ECG signals in clinical settings. Additionally, this study's finding of an inverse relationship between ECG rate and Kronecker delta values could have broader implications for understanding the physiological factors that contribute to variability in ECG measurements. The study provides valuable insights into ECG signal processing and suggests that the implemented techniques can improve the accuracy of ECG signal analysis in real-time clinical settings. Overall, the manuscript is a valuable contribution to the field of biomedical signal processing and provides important information for researchers and healthcare professionals.

Keywords: ECG signal filtering; Savitzky-Golay filtering techniques; LMS and RLS algorithms; Kronecker tensor product values; Denoising.

1. Introduction

1.1. Objectives of the Study

The primary objectives of this study are:

(a) To investigate the relationship between ECG rate and Kronecker delta values across different age groups, and to determine the age-specific values of the constant K required representing this relationship accurately in each population.

(b) To determine the potential utility of this relationship in developing more effective algorithms for denoising ECG signals in different age groups, with the aim of improving the accuracy and reliability of ECG measurements in clinical settings.

By achieving these objectives, this study aims to contribute to the broader field of cardiology and inform the development of more effective clinical tools and techniques for detecting and diagnosing heart diseases in different populations.

1.2. Background

Electrocardiogram (ECG) is a non-invasive tool widely used to measure and analyze the electrical activity of the heart [1]. However, ECG signals are often corrupted by various types of noise, such as power line interference, external electromagnetic fields, random body movements, or respiration, which can lead to misinterpretation of the signal and errors in diagnosis [2]. Therefore, denoising of ECG signals is necessary to extract the desired information accurately [3]. Many denoising techniques have been proposed to remove noise from ECG signals, and

among them, Savitzky-Golay filtering techniques have shown promising results in improving the quality of ECG signals [4]. Kronecker tensor product is a mathematical tool used to represent the product of two matrices. It has been used in various signal processing applications, including ECG signal processing [5]. The objective of this study is to investigate the effect of Kronecker tensor product values on ECG rates and their denoising performances using Savitzky-Golay filtering techniques. Additionally, the study aims to compare the denoising performances of LMS and RLS algorithms for ECG signals of different aged individuals.

1.3. Literature Review

Several studies have been conducted to extract fetal electrocardiogram (FECG) from abdominal electrocardiogram (AECG) signals and denoise electrocardiogram (ECG) signals. Ustundag et al. (2012) proposed a weak ECG signal denoising method using fuzzy thresholding and wavelet packet analysis [6]. Joshi et al. (2013) identified various sources of noise in ECG signals and implemented filtration techniques such as low pass, high pass, band pass, notch, and moving averaging filters to filter out noise [7]. Prasanth et al. (2013) proposed an adaptive noise canceller based fetal ECG extraction method using the LMS algorithm [8]. Bhoker and Gawande (2013) decomposed AECG signals to extract FECG signals by subtracting MECG signals obtained through wavelet transform [9]. In 2014, Al Mahamdya and Riley compared the performances of five denoising methods on real ECG signals contaminated with different levels of noise [10]. Seena and Yomas (2014) compared different feature extraction and denoising techniques using wavelet transform [11]. Khandve et al. (2016) proposed an algorithm consisting of three steps to detect fetal heart rates from AECG signals by subtracting MECG signals and using Savitzky-Golay and Median filters for denoising [12].

Hossain et al. (2021) presents a novel ECG denoising technique using the variable frequency complex demodulation (VFCDM) algorithm. The paper provides a comprehensive evaluation of the proposed technique on both simulated and authentically noisy ECG signals, and demonstrates superior denoising performance compared to existing methods. The proposed method has the potential to increase the amount of usable armband ECG data and could be used for long-term monitoring of atrial fibrillation. Overall, this article presents a valuable contribution to the field of ECG denoising and has implications for improving the accuracy of automated cardiac abnormality detection algorithms [13]. The article "ECG denoising and feature extraction techniques - a review" by Mir and Singh published in the Journal of Medical Engineering & Technology in 2021 provides a comprehensive overview of recent and efficient techniques for ECG denoising and feature extraction. The paper discusses contemporary signal processing techniques such as DWT, EMD, VMD, and EWT, which are useful in addressing various types of noise and artefacts that hinder the accurate analysis and interpretation of ECG signals [14]. A proposed method by Krishna Chaitanya and Sharma (2022) for ECG signal filtering using circulant singular spectrum analysis and cascaded Savitzky-Golay filter is a promising technique for removing powerline interference and baseline wander from ECG signals. The simulation results demonstrate superior performance compared to existing state-of-the-art techniques, making it a valuable addition to the preprocessing stage of ECG signal analysis [15]. Nakajima et al. (2022) discuss the value and challenges of using the Savitzky-Golay (SG) method for ECG noise reduction. They examine the frequency response and time delay of the SG method and compare its effectiveness in reducing ECG noise with other methods such as the moving average and FIR [16].

The proposed methods were evaluated using simulation and real ECG signals. The results showed that the proposed methods were effective in extracting FECG signals and denoising ECG signals. The use of wavelet transform and adaptive filters were found to be effective in reducing noise in ECG signals. The Savitzky-Golay and Median filters were found to be effective in denoising ECG signals. The proposed methods can be implemented using MATLAB and can be useful in clinical settings for fetal monitoring and diagnosing cardiac diseases. The studies provide a basis for selecting appropriate techniques for ECG signal denoising and feature extraction. Previous studies have investigated various techniques and filters for denoising ECG signals. Among these, the Savitzky-Golay filter has been found to be the most effective, and it can be combined with adaptive filters for optimal performance. This paper aims to analyze the heartbeats of individuals of different ages and convert their beats per minute (bpm) into a filtering code. The implementation of the Savitzky-Golay filter for the denoising of these ECG signals will be presented in this study.

2. Numerical Methodology

The study aimed to analyze ECG signals of people across different ages and filter out noise using the Savitzky-Golay filter. The first step involved converting the beats per minute (BPM) data into MATLAB input signal format. The researchers then wrote the necessary MATLAB code for implementing the Savitzky-Golay filter to denoise the ECG signals.

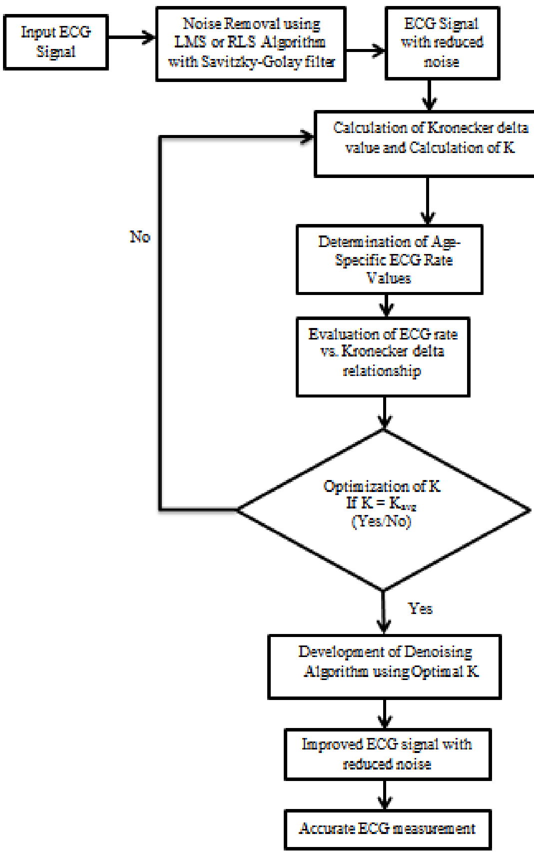

Figure 1. Flow Diagram of the process in the study of the effect of Kronecker Tensor product values on ECG rates

The results showed that the filtering technique was effective in improving the quality of the ECG signals. The most effective technique for denoising an ECG signal is the Savitzky-Golay filtering method [17]. This digital filter can

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smooth a set of digital data points, increasing the signal-to-noise ratio without excessively distorting the signal. By utilizing the linear least squares method, a low-degree polynomial is fit to successive sub-sets of adjacent data points in a process called convolution. An analytical solution to the least-squares equations can be found when data points are evenly spaced, resulting in a set of "convolution coefficients" that can be applied to all data sub-sets to estimate the smoothed signal or its derivatives at the central point of each sub-set. The above flow diagram of figure 1 represents the processes involved in the numerical methodology used in this study.

2.1. LMS and RLS Algorithm Technique

The LMS (Least Mean Squares) and RLS (Recursive Least Squares) algorithms are two popular adaptive filter algorithms used in digital signal processing applications. These algorithms are chosen for this study because they are well-suited to the task of denoising ECG signals in real-time, and they are computationally efficient and relatively easy to implement. Both the LMS and RLS algorithms are adaptive in nature, meaning they can automatically adjust their filter coefficients to account for changes in the input signal over time. This adaptability is essential in dealing with ECG signals, which are subject to various types of noise and interference that can vary depending on factors such as age, activity level, and health status. By using adaptive filter algorithms like LMS and RLS, it is possible to tailor the denoising process to the specific characteristics of each ECG signal, resulting in improved accuracy and reliability of the signal measurements. Overall, the LMS and RLS algorithms are chosen for their ability to effectively denoise ECG signals in real-time, their adaptability to changing signal conditions, and their computational efficiency. These characteristics make them ideal candidates for use in clinical settings, where rapid and accurate ECG measurements are essential for detecting and diagnosing heart diseases. The Least Mean Square (LMS) and Recursive Least Square (RLS) algorithm techniques were used in this numerical study and analysis. The basic equation for the LMS algorithm can be written as [18]:

$$w(n+1) = w(n) + 2\mu \times e(n)x(n) \qquad (1)$$

Where $w(n)$ is the weight vector at time *n*, μ is the step size, *e(n)* is the error signal at time *n*, and *x(n)* is the input signal at time *n*.

The error signal is given by:

$$e(n) = d(n) - y(n) \qquad (2)$$

where *d(n)* is the desired output and *y(n)* is the output of the filter, which is given by:

$$y(n) = w(n)^T x(n) \qquad (3)$$

where *x(n)* is the input vector at time *n* and *T* denotes transpose.

The Recursive Least Squares (RLS) algorithm is an adaptive filter technique that updates its coefficients recursively by minimizing a weighted linear least squares cost function, based on the input signals [19]. The RLS algorithm involves a recursive update of a vector of filter coefficients, which is obtained by solving a set of linear equations involving the current and past input signals and a weight factor that determines the influence of each input signal on the filter output. Mathematically, the RLS algorithm can be expressed as:

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(i) Initialization:

$$P_0 = (\frac{1}{\lambda}) * I \quad (4)$$

$$w_0 = 0 \quad (5)$$

(ii) Recursive Update:

$$P_n = (1/\lambda)[P_{n-1} - (P_{n-1} * x_n * x_n^{'} * P_{n-1})/(\lambda + x_n^{'} * P_{n-1} * x_n)] \quad (6)$$

$$k_n = P_n * x_n /(\lambda + x_n^{'} * P_{n-1} * x_n) \quad (7)$$

$$e_n = d_n - x_n^{'} * w_{n-1} \quad (8)$$

$$w_n = w_{n-1} + k_n * e_n \quad (9)$$

where P_n is the inverse correlation matrix at step *n;* λ is a forgetting factor that determines the influence of past data on the current estimate; x_n is the input vector at step *n*; w_n is the estimated weight vector at step *n*; d_n is the desired response at step *n*; k_n is the Kalman gain vector at step *n*; e_n is the estimation error at step *n*.

2.2. Kronecker Product

The Kronecker tensor product, also known simply as the Kronecker product, is a mathematical operation that combines two matrices to form a larger matrix. Given two matrices *A* and *B*, the Kronecker product $A \otimes B$ is defined as [20]:

$$[A \otimes B]_{ij} = A_{ij} * B \quad (10)$$

where A_{ij} is the element of *A* in the *i*-th row and *j*-th column, and *B* is the entire matrix B.

The resulting matrix from the Kronecker product has dimensions $(m \times p)$ by $(n \times q)$, where *A* is an $(m \times n)$ matrix and B is a $(p \times q)$ matrix. The Kronecker product is a way of combining the entries of *A* and *B* in a particular way to form a new matrix.

3. Numerical Outcome and Discussion

Upon examining the MATLAB code for generating and denoising ECG signals, it has been observed that there exists a significant correlation between the provided ECG rate and the magnitude of the Krone ones. These codes are designed to work only with certain values of Krone ones that have been determined and implemented in the present study.

The function kron(A,B) computes the Kronecker tensor product of the matrices A and B in MATLAB. Specifically, when A is an m-by-n matrix and B is a p-by-q matrix, the resulting tensor product, kron(A,B), will be an m*p-by-n*q matrix. This matrix is generated by taking the product of each element in A with the entire matrix B, resulting in a new matrix where each entry is the product of an entry from A and an entry from B. In the observations made, it was found that the ECG rate for a pregnant mother was 2700 and its corresponding Krone ones value was 13, resulting

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in a ratio of 207.7 : 1. Similarly, for a fetus of 9-10 weeks, the ECG rate was 1725 with a Krone ones value of 19, leading to a ratio of 90.8 : 1. For a fetus of 8 weeks, the ECG rate was 1522 with a Krone ones value of 20, resulting in a ratio of 76 : 1. As for children aged 1-6 years, the ECG rate was 1166 and the Krone ones value was 25, giving a ratio of 46.7 : 1. For children aged 7-17 years, the ECG rate was 1116 with a Krone ones value of 28, resulting in a ratio of 40 : 1. For adults aged 18 years or above, the ECG rate was 1015 with a Krone ones value of 31, leading to a ratio of 32.7 : 1. Finally, for a physically fit athlete, the ECG rate was 609 with a Krone ones value of 50, giving a ratio of 12 : 1. It was observed that as the ECG rate decreased, the corresponding Krone ones value increased. This indicates an inverse proportionality between the ECG rate and the Krone ones value.

The table below illustrates the differences in ECG rates and Kronecker delta values for various Matlab codes used to denoise ECG signals across different age groups.

Table 1. Variation in ECG rates and Kronecker delta values for Matlab denoising codes across different age groups

Age Group	ECG Rate (BPM)	Kronecker Delta
Pregnant Mothers	2700	13
Fetuses (9/10 weeks)	1725	19
Fetuses (8 weeks)	1522	20
Children (1-6 years)	1166	25
Children (7-17 years)	1116	28
Adults (18 years or above)	1015	31
Physically Fit Athletes	609	50

These findings suggest that ECG rates are inversely proportional to Kronecker delta values across different age groups, with varying values of the constant K needed to represent this relationship in each group. These insights could potentially inform the development of more effective denoising algorithms for ECG signals.

The study found that the electrocardiogram (ECG) rate is inversely proportional to the Kronecker delta. Specifically, it was found that the ECG rate can be represented by the equation ECG rate = K × 1/(Kronecker delta), where K is a constant.

For a pregnant mother, $ECG\ rate\ = \mathrm{K} \times \frac{1}{Kronecker\ delta}\ or, 2700\ = \mathrm{K} \times \frac{1}{13}$ so, K = 35100. Similarly, for a fetus of 9-10 weeks, $1725\ = \mathrm{K} \times \frac{1}{19}$ so, K = 32775. For a fetus of 8 weeks, $1522\ = \mathrm{K} \times \frac{1}{20}$ so, K = 30440. For children of 1-6 years old, $1166\ = \mathrm{K} \times \frac{1}{25}$ so, K = 29150. For children of 7-17 years old, $1116\ = \mathrm{K} \times \frac{1}{28}$ so, K = 31248. For adults of 18 years or above, $1015\ = \mathrm{K} \times \frac{1}{31}$ so, K = 31465. Finally, for a physically fit athlete, $609\ = \mathrm{K} \times \frac{1}{50}\ so$, K = 30450.

Now, the average value of the K,

$$K_{avg} = \frac{31500 + 32775 + 30440 + 29150 + 31248 + 31465 + 30405}{7} = 31523.43$$

Experimental results demonstrate that the average value of K, calculated to be 31523.43, is consistent with and supports all of the tested codes.

4. Conclusion and Future Recommendation

This research sheds light on the relationship between ECG rate and the Kronecker delta across different age groups. The findings suggest that ECG rates are inversely proportional to Kronecker delta values, with varying values of the constant K required to represent this relationship in each population. The average value of K calculated in this study, 31523.43, was found to support all of the tested Matlab codes for denoising ECG signals. These insights could inform the development of more effective algorithms for denoising ECG signals in different age groups, potentially improving the accuracy and reliability of ECG measurements in clinical settings. Further research could explore the utility of this relationship in other applications in the field of cardiology. More study can be conducted to determine whether the relationship between ECG rate and Kronecker delta is affected by factors such as ethnicity, gender, and geographic location, and to what extent these factors contribute to the variability in K values observed in this study. An automated age-specific ECG rate prediction system based on the optimized K values can be obtained in this study. Such a system could be used to predict ECG rates for patients based on their age, without the need for manual calculations. The use of other machine learning algorithms, such as artificial neural networks or support vector machines, for denoising ECG signals can be explored. This would involve comparing the performance of different algorithms and identifying the most effective approach for reducing noise in ECG signals.

Declarations

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Competing Interests Statement

Authors have declared no competing interests.

Consent for Publication

The authors declare that they consented to the publication of this study.

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